Global Stability Analysis: Cylinder Flow

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Introduction

This tutorial further explores the use of the spectral/hp element framework *Nektar++* to perform global stability computations. Information on how to install the libraries, solvers, and utilities on your own computer is available on the webpage www.nektar.info.

This tutorial assumes the reader has already completed the previous tutorial in the Flow Stability series on the channel and therefore already has the necessary software installed.

> **Task 1.1**
>
> Prepare for the tutorial. Make sure that you have:
>
> - Downloaded the tutorial files: `http://doc.nektar.info/tutorials/5.4.0/flow-stability/cylinder/flow-stability-cylinder.tar.gz` Unpack it using `unzip flow-stability-cylinder.tar.gz` to produce a directory `flow-stability-cylinder` with subdirectories called `tutorial` and `complete` We will refer to the `tutorial` directory as `$NEKTUTORIAL`.

In this tutorial we will compute the direct and adjoint modes of a two-dimensional flow past a cylinder. We will investigate a case in the subcritical regime ($Re = 42$), below the onset of the Bernard-von Kärmän vortex shedding that is observed when the Reynolds number is above the critical value $Re_c \simeq 47$; this analysis is important because it allows us to study the sensitivity of the flow, much like that reported by Giannetti and Luchini (J. Fluid Mech., 2007; **592**:177-194). Due to the more complex nature of the flow and the more demanding computational time that is required, only some basic information will be presented in this section, mainly to show the potential of the code for stability analysis.

The files contained in the `$NEKTUTORIAL` directory are as follows:

- Folder `geometry`

  – `Cylinder.geo` - *Gmsh* file that contains the geometry of the problem
  – `Cylinder.msh` - *Gmsh* generated mesh data listing mesh vertices and elements.

- Folder `base`

  – `Cylinder-Base.xml` - *Nektar++* session file, generated with the `$NEK/NekMesh` utility, for computing the base flow.
  – `Cylinder-Base.fld` - *Nektar++* field file that contains the base flow, generated using
    `Cylinder-Base.xml`.

- Folder `stability/Direct`

  – `Cylinder_Direct.xml` - *Nektar++* session file, generated with `$NEK/NekMesh`.
  – `Cylinder_Direct.bse` - *Nektar++* field file that contains the base flow.
  – `Cylinder_Direct.rst` - *Nektar++* field file that contains the initial conditions.

- Folder `stability/Adjoint`

  – `Cylinder_Adjoint.xml` - *Nektar++* session file, generated with `$NEK/NekMesh`.
  – `Cylinder_Adjoint.bse` - *Nektar++* field file that contains the base flow.
  – `Cylinder_Adjoint.rst` - *Nektar++* field file that contains the initial conditions.

- Folder `stability/Receptivity` Pre-computed output files for calculating the receptivity.

The mesh is shown in figure 1.1 along with a detailed view around the cylinder. This mesh is made up of 782 quadrilateral elements.

Note: It is important to note that stability and transient growth calculations in particular, have a strong dependence on the domain size as reported by Cantwell and Barkley (Physical Review E, 2010; **82**); moreover, poor mesh design can lead to incorrect results. Specifically, the mesh must be sufficiently refined around the cylinder in order to capture the separation of the flow and abrupt variations in the size of the elements should be avoided.
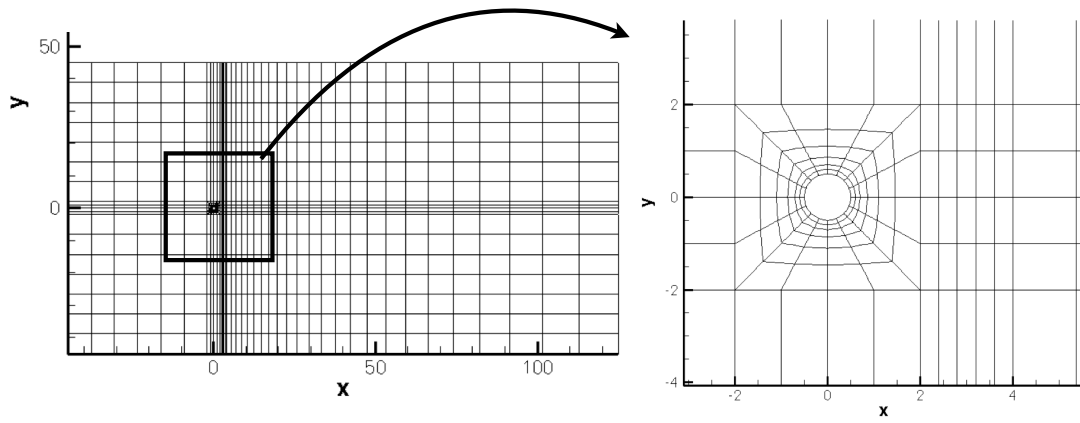
Figure 1.1 Mesh used for the direct stability analysis

Computation of the base flow

`Cylinder-Base.xml` can be found inside the `$NEKTUTORIAL/base` folder. This is the *Nektar++* file generated using `$NEK/NekMesh` and augmented with all the configuration settings that are required. In this case, CFL conditions can be particularly restrictive and the time step must be set around 8×10^{-4}. We will be using Reynolds number $Re = 42$ for this study.

The supplied file `Cylinder-Base.bse` is the converged base flow required for the analysis and is the result of running `Cylinder-Base.xml`. To have a steady solution it was necessary to evolve the fields for a non-dimensional time $\tau \geq 300$ and it is very important to be sure that the solution is steady. This can be verified by putting several history points on the centre line of the flow and monitoring their variation.

> **Task 2.1**
> Convert the base flow into VTK format and visualise the profile of the flow past a cylinder in *Paraview* .

The base flow should look like the one in figure 2.1.
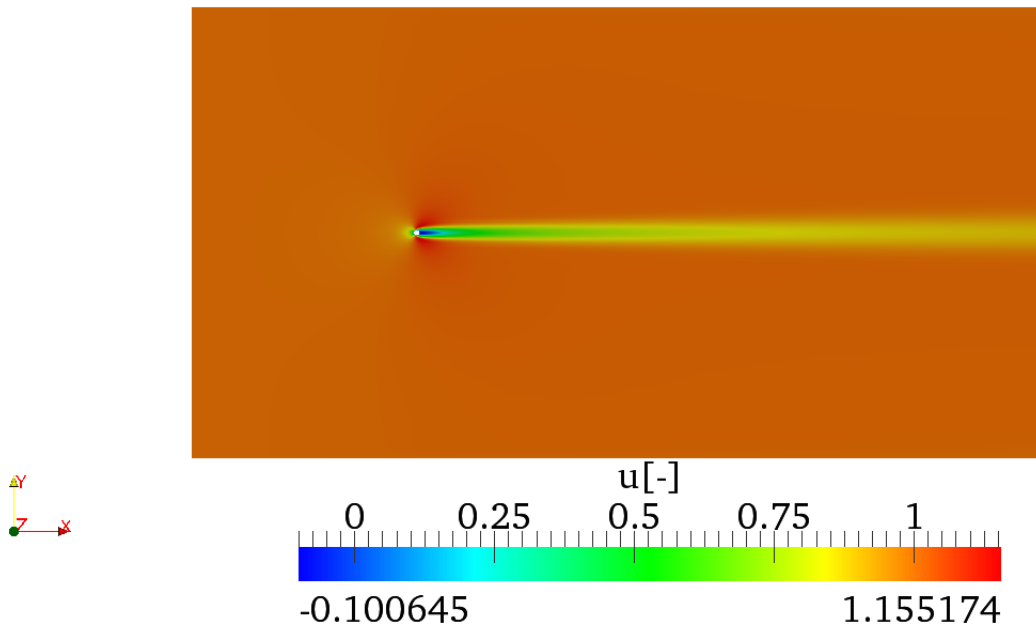
Figure 2.1 Base flow for the cylinder test case

Stability analysis

3.1 Direct

In the folder `$NEKTUTORIAL/stability/Direct` there are the files that are required for the direct stability analysis. Since, the computation would normally take several hours to converge, we will use a restart file and a Krylov-space of just $\kappa = 4$. Therefore, it will be possible to obtain the eigenvalue and the corresponding eigenmode after 2 iterations.

> **Task 3.1**
>
> Define a Kyrlov space of 4 and compute the leading 2 eigenvalues and the eigenvectors of the problem using Arpack and the restart file `Cylinder_Direct.rst`.

The simulation should converge in 6 iterations and the terminal screen should look similar to the one below:

```
==========================================================================
            EquationType: UnsteadyNavierStokes
            Session Name: Cylinder_Direct
            Spatial Dim.: 2
        Max SEM Exp. Order: 7
           Expansion Dim.: 2
         Projection Type: Continuous Galerkin
               Advection: explicit
               Diffusion: explicit
               Time Step: 0.0008
             No. of Steps: 1250
       Checkpoints (steps): 1000
         Integration Type: IMEXOrder2
==========================================================================
        Arnoldi solver type    : Arpack
        Arpack problem type    : LM
        Single Fourier mode    : false
```

```
        Beta set to Zero        : false
        Evolution operator      : Direct
        Krylov-space dimension  : 4
        Number of vectors       : 2
        Max iterations          : 500
        Eigenvalue tolerance    : 1e-06
==========================================================
Initial Conditions:
  - Field u: from file Cylinder_Direct.rst
  - Field v: from file Cylinder_Direct.rst
  - Field p: from file Cylinder_Direct.rst
Writing: "Cylinder_Direct_0.chk"
        Inital vector          : input file
Iteration 0, output: 0, ido=1 Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 1          CPU Time: 46.5477s
Time-integration  : 46.5477s
Writing: "Cylinder_Direct.fld"
Iteration 1, output: 0, ido=1 Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 2          CPU Time: 41.7221s
Time-integration  : 41.7221s
Iteration 2, output: 0, ido=1 Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 3          CPU Time: 41.8717s
Time-integration  : 41.8717s
Iteration 3, output: 0, ido=1 Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 4          CPU Time: 41.9465s
Time-integration  : 41.9465s
Iteration 4, output: 0, ido=1 Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 5          CPU Time: 41.987s
Time-integration  : 41.987s
Iteration 5, output: 0, ido=1
Writing: "Cylinder_Direct_1.chk"
Steps: 1250     Time: 6          CPU Time: 42.2642s
Time-integration  : 42.2642s
Iteration 6, output: 0, ido=99
Converged in 6 iterations
Converged Eigenvalues: 2
        Magnitude   Angle      Growth       Frequency
EV: 0 0.9792      0.726586   -0.0210196  0.726586
Writing: "Cylinder_Direct_eig_0.fld"
EV: 1 0.9792      -0.726586  -0.0210196  -0.726586
Writing: "Cylinder_Direct_eig_1.fld"
L 2 error (variable u) : 0.0501837
L inf error (variable u) : 0.0296123
L 2 error (variable v) : 0.0635524
L inf error (variable v) : 0.0355673
L 2 error (variable p) : 0.0344665
L inf error (variable p) : 0.0176009
```

Task 3.2

Verify that the leading eigenvalues show a growth rate of $\sigma = -2.10196 \times 10^{-2}$ and a frequency $\omega = \pm 7.26586 \times 10^{-1}$.

Task 3.3

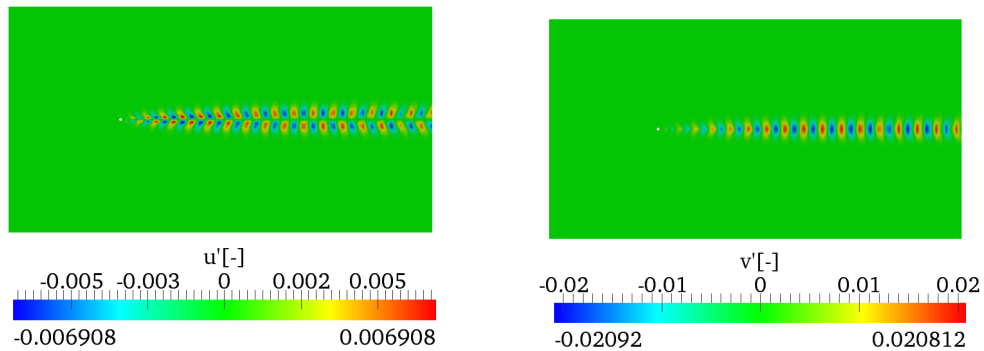Plot the leading eigenvector in *Paraview* or VisIt. This should look like the solution shown in figures 3.1.

Figure 3.1 u'-component and v'-component of the eigenmode

3.2   Adjoint

After the direct stability analysis, it is now interesting to compute the eigenvalues and eigenvectors of the adjoint operator $\mathcal{A}^*$ that allows us to evaluate the effects of generic initial conditions and forcing terms on the asymptotic behaviour of the solution of the linearised equations. In the folder `Cylinder/Stability/Adjoint` there is the file `Cylinder_Adjoint.xml` that is used for the adjoint analysis.

Task 3.4

Set the `EvolutionOperator` to `Adjoint`, the Krylov space to 4 and compute the leading eigenvalue and eigenmode of the adjoint operator, using the restart file `Cylinder_Adjoint.rst`

The solution should converge after 4 iterations and the terminal screen should look like this:

```
==========================================================================
             EquationType: UnsteadyNavierStokes
             Session Name: Cylinder_Adjoint
             Spatial Dim.: 2
        Max SEM Exp. Order: 7
```

```
              Expansion Dim.: 2
            Projection Type: Continuous Galerkin
                  Advection: explicit
                  Diffusion: explicit
                  Time Step: 0.001
              No. of Steps: 1000
        Checkpoints (steps): 1000
           Integration Type: IMEXOrder3
=======================================================================
      Arnoldi solver type    : Arpack
      Arpack problem type    : LM
      Single Fourier mode    : false
      Beta set to Zero       : false
      Evolution operator     : Adjoint
      Krylov-space dimension : 4
      Number of vectors      : 2
      Max iterations         : 500
      Eigenvalue tolerance   : 0.001
======================================================
Initial Conditions:
Field p not found.
  - Field u: from file Cylinder_Adjoint.rst
  - Field v: from file Cylinder_Adjoint.rst
  - Field p: from file Cylinder_Adjoint.rst
Writing: "Cylinder_Adjoint_0.chk"
      Inital vector        : input file
Iteration 0, output: 0, ido=1 Steps: 1000    Time: 27    CPU Time: 42.0192s
Writing: "Cylinder_Adjoint_1.chk"
Time-integration  : 42.0192s


Writing: "Cylinder_Adjoint.fld"
Iteration 1, output: 0, ido=1 Steps: 1000    Time: 28    CPU Time: 37.1084s
Writing: "Cylinder_Adjoint_1.chk"
Time-integration  : 37.1084s
Iteration 2, output: 0, ido=1 Steps: 1000    Time: 29    CPU Time: 37.4794s
Writing: "Cylinder_Adjoint_1.chk"
Time-integration  : 37.4794s
Iteration 3, output: 0, ido=1 Steps: 1000    Time: 30    CPU Time: 37.3142s
Writing: "Cylinder_Adjoint_1.chk"
Time-integration  : 37.3142s
Iteration 4, output: 0, ido=99
Converged in 4 iterations
Converged Eigenvalues: 2
        Magnitude    Angle       Growth      Frequency
EV: 0 0.980493    0.727526    -0.0197     0.727526
Writing: "Cylinder_Adjoint_eig_0.fld"
EV: 1 0.980493    -0.727526   -0.0197     -0.727526
Writing: "Cylinder_Adjoint_eig_1.fld"
L 2 error (variable u) : 0.434746
L inf error (variable u) : 0.156905
```

```
L 2 error (variable v) : 0.698425
L inf error (variable v) : 0.120624
L 2 error (variable p) : 0.216948
L inf error (variable p) : 0.0676028
```

Task 3.5

Verify that the eigenvalues of the system are $\lambda_{1,2} = 0.980495 \times e^{\pm i0.727502}$ with a growth rate equal to $\sigma = -1.969727 \times 10^{-2}$ and a frequency $\omega = \pm 7.275024 \times 10^{-1}$.

Task 3.6

Plot the leading eigenmode in *Paraview* or VisIt that should look like figures 3.2 and 3.2.

Note that, in spatially developing flows, the eigenmodes of the direct stability operator tend to be located far downstream while the eigenmodes of the adjoint operator tend to be located upstream and near to the body, as can be seen in figures 3.2 and 3.3. From the profiles of the eigemodes, it can be deducted that the regions with the maximum receptivity for the momentum forcing and mass injection are near the wake of the cylinder, close to the upper and lower sides of the body surface, in accordance with results reported in the literature.
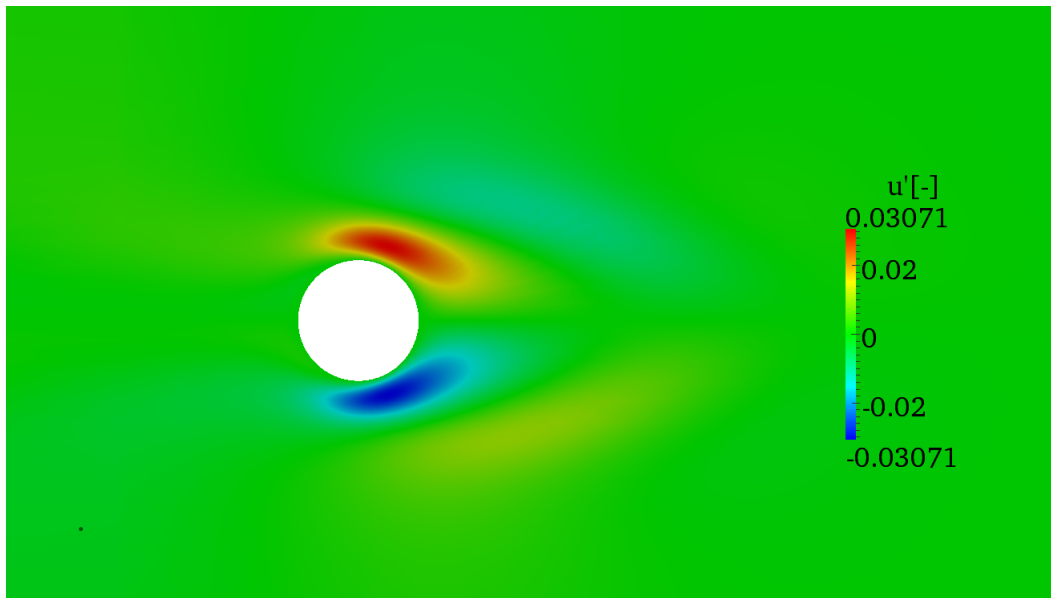
Figure 3.2 Close-up of the u^*-component of the adjoint eigenmode.
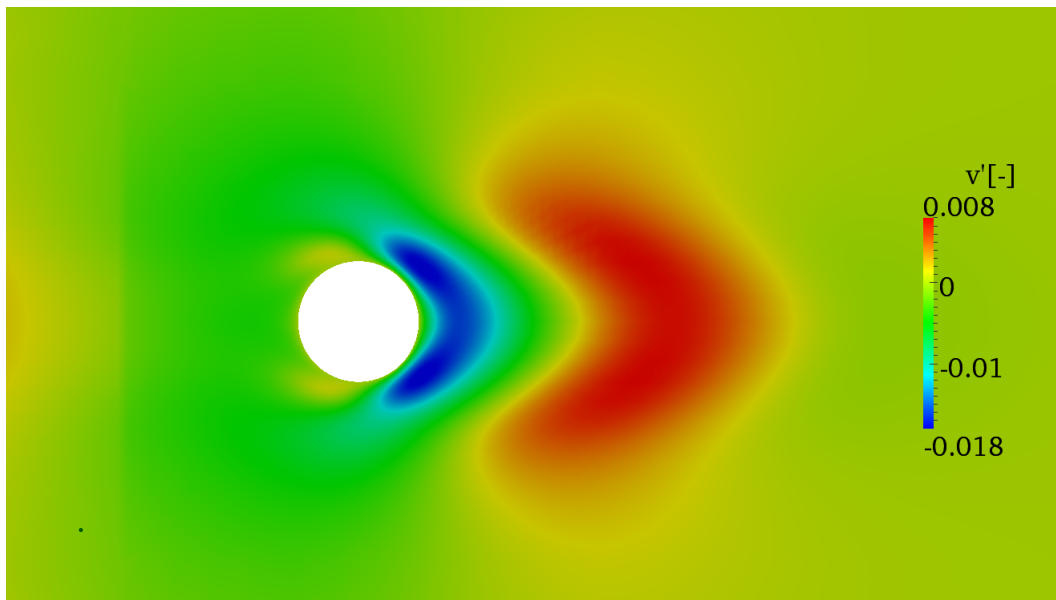
Figure 3.3 The v^*-component of the adjoint eigenmode extends far upstream of the cylinder